APPLICATION OF COHN'S SENSITIVITY THEOREM TO TIME DOMAIN RESPONSES.

by

Alan Lee Dahlvig

United States
Naval Postgraduate School

THESIS

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Alan Lee Dahlvig

Lieutenant, United States Navy
B.S.E.E., University of Oklahoma, 1963

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ABSTRACT

A sensitivity theorem by R. M. Cohn states that for linear resistive circuits the ratio of a fractional change in the d.c. input resistance to a fractional change in an internal resistance is equal to the square of the ratio of the current through that internal resistance to the d.c. input current. The theorem is extended to show the sensitivity of input impedance to changes in internal impedances for an arbitrary network at all frequencies. Equations are developed which show the relation between sensitivities and instantaneous power in the frequency domain. An extension to the time domain makes digital computer solutions possible.

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I. INTRODUCTION

In 1950, R. M. Cohn [Ref. 1] presented a linear resistive network sensitivity theorem which was new and significant in its approach to sensitivity. Although the theorem is restrictive in its original form, it is, in fact, a particular case of a more general result involving circuit sensitivities as shall be discussed here.

In Chapter 2, a proof of Cohn's Theorem is presented. In Chapter 3, the theorem is extended to an arbitrary input impedance. In Chapter 4, the results of the previous chapter, which were developed in the frequency domain, are interpreted in terms of time domain responses. A general relationship between sensitivity and power relationships is developed. The results are then illustrated by means of an example for which a computer program has been written to obtain the appropriate waveforms.

II. PROOF OF COHN'S THEOREM

Consider a two terminal resistive network consisting of branches $b_1, b_2, \ldots\ldots, b_n$, having independendent variable resistors, $r_1, r_2, \ldots, r_n$. The input resistance R (see Figure 1) is then a rational function of the r's. Consider also, that a unit current is applied to the input terminals. It can then be shown that R has the following property:

$$\frac{\partial R}{\partial r_m} = i_m^2 \qquad \text{and} \qquad \frac{\partial R/R}{\partial r_m/r_m} = \frac{i_m^2 r_m}{R} = \frac{P_{r_m}}{P_R} \tag{1}$$

where i_m is the current flowing in b_m. Equation (1), which is Cohn's Theorem [Ref. 1], states that the ratio of the fractional change in the input resistance to the fractional change in an internal resistance is proportional to the ratio of the power dissipated in the internal resistance to the input power. The proof that follows is based on that of Cohn.

Let i_j denote the current flowing in, and v_j the voltage across, branch b_j. Let the network consist of q nodes designated by $n_1, n_2, \ldots, n_q$ with the current source connected from n_1 to n_q. If the loops are designated by $l_1, l_2, \ldots, l_t$ and the direction of current flow in each branch and in each loop assigned arbitrarily, then Kirchoff's equations may be stated as:

$$\sum_{j=1}^{t} b_{ij} v_j = 0 \qquad i = 1,2,\ldots,t \tag{2}$$

$$\sum_{j=1}^{q} a_{ij} i_j = g_j \qquad i = 1,2,\ldots,q \tag{3}$$

b_{ij} = 1 if the branch current and loop current have the same direction,

= -1 if the branch current and loop current have opposite directions,

= 0 if the branch is not in the loop.

a_{ij} = 1 if the branch current is directed away from the node,

= -1 if the branch current is directed toward the node,

$= 0$ if the branch is not connected to the node.

$g_j = 1$ for $j = 1$

$= -1$ for $j = q$

$= 0$ elsewhere.

Define $e_j = i_j r_j$ to be the voltage across b_j in a positive direction and let the voltage of node n_i with respect to node n_j be $E_{ij} = f_j e_j$ with $f_j = 1$ if i_j goes from n_i to n_j and $f_j = -1$ if i_j goes from n_j to n_i. If node q is grounded, $E_{1q} = E_1 = R$. (4)

Since Kirchoff's laws imply the conservation of power, the power input must equal the sum of the powers consumed by the individual branches. This may be written as:

$$E_1 I = \sum_{j=1}^{n} i_j^2 r_j = R \qquad . \tag{5}$$

If equation (3) is differentiated with respect to r_m, it is found that:

$$\frac{\partial}{\partial r_m} = \left[\sum_{j=1}^{n} a_{ij} i_j \right] = \sum_{j=1}^{n} a_{ij} \frac{\partial i_j}{\partial r_m} = \frac{\partial g_j}{\partial r_m} = 0 \qquad i = 1,2,\ldots,q \tag{6}$$

Let the nodes of b_j be denoted by n_{ja} and n_{jb} such that e_j is positive going from n_{ja} to n_{jb}, then

$$i_j^2 r_j = e_j i_j = (E_{jb} - E_{ja}) i_j \tag{7}$$

$$\sum_{j=1}^{n} i_j^2 r_j = \sum_{j=1}^{n} (E_{jb} - E_{ja}) i_j \qquad . \tag{8}$$

By defining a function h_j (an arbitrary function of the r's) such that

$$\sum_{j=1}^{n} a_{ij} h_j = 0 \qquad\qquad i = 1,2,\ldots.,q \tag{9}$$

and using equation (7) above,

$$\sum_{j=1}^{n} i_j r_j h_j = \sum_{j=1}^{n} e_j h_j = \sum_{j=1}^{n} (E_{jb} - E_{ja}) h_j \quad . \tag{10}$$

Since E_{jb} and E_{ja} are voltages measured with respect to node q, they may be written as E_i's. Also, a network is not necessarily a series connection of resistors. Thus, E_{jb} and E_{ja} may be associated with more than one branch. If the sum on the right hand side of equation (8) is performed and all terms with the same E_i are combined, it is seen that:

$$\sum_{j=1}^{n} i_j r_j h_j = \sum_{j=1}^{n} (E_{jb} - E_{ja})\, h_j = \sum_{i=1}^{q} \left(\sum_{j=1}^{n} a_{ij} h_j \right) E_i \tag{11}$$

$$= 0 \text{ by the definition of } \sum a_{ij} h_j \quad .$$

This can easily be shown in a simple example (see Figure 2).

$$\sum_{k=1}^{q} (E_{kb} - E_{ka}) h_k = (E_1 - E_2) h_1 + (E_1 - E_3) h_2 + (E_1 - E_2) h_5 +$$

$$(E_2 - E_3) h_4 + (E_3 - E_q) h_3$$

$$= E_1 (h_1 + h_2 + h_5) + E_2 (h_4 - h_5) + E_3 (-h_2 + h_3 - h_4) +$$

$$E_q (-h_1 - h_3) \quad . \tag{12}$$

From the definition for a_{ij} given earlier:

$$a_{11} = a_{12} = a_{15} = a_{24} = a_{33} = 1,$$

$$a_{25} = a_{34} = a_{32} = a_{q1} = a_{q3} = -1,$$

and all others equal 0.

If each h_j in equation (12) is multiplied by its appropriate a_{ij} such that i corresponds to the appropriate index of the E_i, it will be seen that equation (11) holds.

Comparing equation (6) and the defining equation for h_j, (9), it is seen that $\partial i_j / \partial r_m = h_j$. Thus from (11)

$$\sum_{j=1}^{n} i_j r_j h_j = \sum_{j=1}^{n} r_j i_j \frac{\partial i_j}{\partial r_m} = 0 \quad . \qquad (13)$$

If equation (5) is now differentiated with respect to r_m, the result is:

$$\frac{\partial R}{\partial r_m} = \frac{\partial}{\partial r_m}\left[\sum_{j=1}^{n} i_j^2 r_j\right] = \sum_{j=1}^{n} i_j^2 \frac{\partial r_j}{\partial r_m} + 2\sum_{j=1}^{n} i_j r_j \frac{\partial i_j}{\partial r_m} \qquad (14)$$

From equation (13)

$$2\sum_{j=1}^{n} i_j r_j \frac{\partial i_j}{\partial r_m} = 0 \quad .$$

Since the resistors of the circuit are independent,

$$\frac{\partial r_j}{\partial r_m} = \begin{cases} 0, & j \neq m \\ 1, & j = m \end{cases} \quad ,$$

and equation (14) becomes

$$\frac{\partial R}{\partial r_m} = i_m^2 \quad . \qquad (14a)$$

This is Cohn's theorem for a resistive network with a unit step input current.

III. EXTENSION OF COHN'S THEOREM TO AN ARBITRARY IMPEDANCE

The expression developed in equation (14a) is interesting, but is limited to purely resistive circuits. Therefore, an extension of equation (14a) to an arbitrary impedance is desireable. By applying a theorem of Tellegen [Ref. 2], it can be shown that:

$$\frac{\partial Z(s)}{\partial Z_b(s)} = \left[\frac{I_b(s)}{I(s)}\right]^2 \tag{15}$$

The method used follows to some extent the work of Penfield [Ref. 3].

Tellegen's theorem states, "In a network configuration, imagine branch currents i such that for every node $\sum i = 0$, imagine branch voltages v such that for every mesh $\sum v = 0$, and for every branch let the positive direction of current be from the + to the - denoting the positive polarity of the voltage. Then $\sum iv = 0$, where the summation is over all branches". Tellegen then goes on to show how for Figure (1) the theorem can be written as $\sum_t i_t v_t = \sum_b i_b v_b$ where i_t and v_t are the terminal voltages and currents and i_b and v_b are branch currents and voltages. It is significant to note that Tellegen's Theorem is strictly topological and is related in no way to the linearity or nonlinearity of the circuit components. For a constant, linear system, Tellegen shows that $\sum I_b V_b' = \sum I_b' V_b$. The unprimed quantities are the first state values and the primed quantities are the second state values of two circuits with the same topology.[1]

Since the Laplace operator is a linear operator, it can be applied and Tellegen's theorem can be written as:

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[1] It is unfortunate that Penfield chose to use the term "state values" since it can easily be confused with the "state variables" often used in circuit analysis. As used here, "state values" refer to the values of the voltages and currents of the branches of two different circuits. These two circuits have the same topology but not necessarily the same circuit elements.

$$\sum_t I_t(s)V_t'(s) = \sum_b I_b(s)V_b'(s) \tag{16}$$

or

$$\sum_t I_t'(s)V_t(s) = \sum_b V_b(s)I_b'(s) \quad . \tag{17}$$

Subtracting (17) from (16) yields:

$$\sum_t \left[I_t(s)V_t'(s) - I_t'(s)V_t(s)\right] = \sum_b \left[V_b'(s)I_b(s) - V_b(s)I_b'(s)\right] \quad . \tag{18}$$

Unless otherwise stated, all calculation in the remainder of this paper will be in the Laplace domain (s - domain).

Consider circuits as shown in Figure 3.

	For circuit (a)		For circuit (b)	
	V(s)	I(s)	V(s)	I(s)
Port	$E_g(s)$	$I(s)$	$E_g(s)$	$I(s) + \delta I(s)$
Circuit	$V_b(s)$	$I_b(s)$	$V_b(s) + \delta V_b(s)$	$I_b(s) + \delta I_b(s)$

Using equation (18),

$$I(s)E_g(s) - [I(s) + \delta I(s)]E_g(s) = \sum \Big\{I_b(s)[V_b(s) + \delta V_b(s)] - V_b(s)[I_b(s) + \delta I_b(s)]\Big\}$$

$$E_g(s)I(s) = \sum_b \Big\{V_b(s)\delta I_b(s) - I_b(s)\delta V_b(s)\Big\}.$$

$V_b(s) = Z_b(s)I_b(s)$ and for small changes

$$\delta V_b(s) = Z_b(s)\delta I_b(s) + I_b(s)\delta Z_b(s) \quad . \tag{20}$$

Multiplying (20) by $I_b(s)$ and rearranging will yield:

$$I_b(s)\delta V_b(s) - V_b(s)\delta I_b(s) = I_b^2(s)\delta Z_b(s) \quad . \tag{21}$$

From the two states in Figure 3, it can be seen that:

$$E_g(s) = I(s)Z(s) \tag{22}$$

and

$$E_g(s) = [I(s) + \delta I(s)][Z(s) + \delta Z(s)]$$

$$= I(s)Z(s) + I(s)\delta Z(s) + Z(s)\delta I(s) + \delta I(s)\delta Z(s) \tag{23}$$

Substituting equation (22) into (23) shows that:

$$I(s)\delta Z(s) + Z(s)\delta I(s) + \delta I(s)\delta Z(s) = 0 \tag{24}$$

$$I^2(s)\delta Z(s) = -E_g(s)\delta I(s) - I(s)\delta I(s)\delta Z(s)$$

$$= \delta I(s)[E_g(s) + I(s)\delta Z(s)] \quad . \tag{25}$$

If $I(s)\delta Z(s) << E_g(s)$ [i.e. $\delta Z(s) << Z(s)$], then:

$$I^2(s)\delta Z(s) = -E_g(s)\delta I(s) \quad . \tag{26}$$

Substituting (26) and (21) into (19), it is found that:

$$I^2(s)\delta Z(s) = \sum_b I_b^2(s)\delta Z_b(s) \quad . \tag{27}$$

Since $Z(s)$ is a function of $Z_b(s)$, $\delta Z(s) = \sum\limits_b \frac{\partial Z(s)}{\partial Z_b(s)} \delta Z_b(s)$.

Putting this result into (27) results in

$$I^2(s) \sum_b \frac{\partial Z(s)}{\partial Z_b(s)} \delta Z_b(s) = \sum_b I_b^2(s)\delta Z_b(s)$$

$$\sum_b \frac{\partial Z(s)}{\partial Z_b(s)} \delta Z_b(s) = \sum_b \left[\frac{I_b(s)}{I(s)}\right]^2 \delta Z_b(s)$$

$$\frac{\partial Z(s)}{\partial Z_b(s)} = \left[\frac{I_b(s)}{I(s)}\right]^2 \quad . \tag{28}$$

This is Cohn's theorem in the s - domain for any impedance.

If s is set equal to $j\omega$, the results can be obtained in the frequency domain:

$$\frac{Z(j\omega)}{Z_b(j\omega)} = \left[\frac{I_b(j\omega)}{I(j\omega)}\right]^2 \quad . \tag{29}$$

The right hand side of (29) may be written as a transfer function as a magnitude and a phase angle.

$$\frac{I_b(j\omega)}{I(j\omega)} = T(\omega)e^{j\theta(\omega)} \tag{30}$$

$$\frac{\partial Z(j\omega)}{\partial Z_b(j\omega)} = \left[\frac{I_b(j\omega)}{I(j\omega)}\right]^2 = T^2(\omega)\underline{/2\theta(\omega)} \quad . \tag{31}$$

If $Z_b(j\omega) = R$,

$$\frac{\partial Z(j\omega)}{\partial R} = T^2(\omega)\underline{/2\theta(\omega)} \quad . \tag{32}$$

If $Z_b(j\omega) = j\omega L$,

$$\frac{\partial Z(j\omega)}{\partial(j\omega L)} = T^2(\omega)\underline{/2\theta(\omega)}$$

and

$$\frac{\partial Z(j\omega)}{\partial L} = j\omega T^2(\omega)\underline{/2\theta(\omega)} = \omega T^2(\omega)\underline{/2\theta(\omega) + \pi/2} \quad . \qquad (33)$$

If $Z_b(j\omega) = 1/j\omega C$,

$$\frac{\partial Z(j\omega)}{\partial(1/j\omega C)} = T^2(\omega)\underline{/2\theta(\omega)}$$

and

$$\frac{\partial Z(j\omega)}{\partial(1/C)} = (1/j\omega)T^2(\omega)\underline{/2\theta(\omega)} = \frac{T^2(\omega)}{\omega}\ \underline{/2\theta(\omega) - \pi/2} \quad . \qquad (34)$$

Thus, the magnitude of the sensitivity changes as the product of $\omega T^2(\omega)$ or $T^2(\omega)/\omega$ and its phase is advanced or retarded by $\pi/2$ when the element of interest is an inductor or a capacitor rather than a resistor.

IV. COHN'S THEOREM IN THE TIME DOMAIN

Equation (28) expresses Cohn's theorem in the s - domain for any input current I(s) applied to an impedance Z(s).

$$\frac{\partial Z(s)}{\partial Z_b(s)} = \left[\frac{I_b(s)}{I(s)}\right]^2$$

This equation is easy to use if the input current I, and the internal element current I_b are known as functions of s. The input current wave shape will be known for most application and its transform to the s - domain will usually not be difficult. However, for any but the most elementary of circuits, the internal element current $I_b(s)$ can be found only through a series of tedious calculations. In contrast, there are several good computer programs which will calculate $i_b(t)$. It is, therefore, desireable to find a direct means of determining the sensitivity as a function of time rather than by taking the inverse transform of (28). For the class of input currents given by (39), as derived later, it can be shown that:

$$\frac{\partial Z(s)}{\partial Z_b(s)} = \sum_{k=0}^{2m} a_k s^{k-2n} I_b^2(s) \qquad (35)$$

where n and m are integers defined by (39). This leads to the following results in the time domain:

$$\frac{\partial z}{\partial x_b} = \sum_{k=0}^{2n} a_k \frac{\partial^{k-2n+j}}{\partial t^{k-2n+j}} [i_b(t) * i_b(t)] \qquad (36)$$

where the symbol * is used to indicate convolution, and

$$\begin{aligned} j &= +1 \text{ if } x_b \text{ is an inductor (L)} \\ &= 0 \text{ if } x_b \text{ is a resistor (R)} \\ &= -1 \text{ if } x_b \text{ is 1/capacitor (1/C)} \end{aligned}$$

A very interesting simplification occurs when $i_g(t) = \delta(t)$, a unit impulse

so that $E(s) = Z(s)$.

$$\frac{\partial E(s)}{\partial Z_b(s)/Z_b(s)} = E_b(s) I_b(s) \tag{37}$$

$$\frac{\partial e(t)}{\partial z_b/z_b} = e_b(t) * i_b(t) \tag{38}$$

Equation (37) shows that the sensitivity is related to the instantaneous power in the frequency domain.

In attempting to develop the time domain equivalent of equation (35), it would appear to a casual observer that a simple application of the principal of convolution would apply. That is, $X(s)Y(s) = \mathcal{L}[x(t) * y(t)]$ and thus, $X(s)/Y(s) = \mathcal{L}[x(t) * 1/y(t)]$. However, this is not true since $\mathcal{L}[y^{-1}(t)] \neq Y^{-1}(s)$. For example:

Let $Y(s) = s$

$y(t)$ = a unit doublet

Now let $1/Y(s) = 1/s$

$1/s$ transforms to a unit step in the time domain and a unit step is not the reciprocal of the unit doublet.

It would appear useful to try to determine if there is a direct way to obtain the inverse transform of $1/Y(s)$ to get a time function which can be convolved with $x(t)$. There does not appear to be any such simple operation.

An alternate possibility is to limit the consideration to input current wave shapes which can be manipulated easily so as to yield a direct solution. Consider an input current of the following form:

$$I(s) = \frac{gs^n}{\sum_{j=0}^{m} b_j s^j} = \frac{N(s)}{D(s)} \tag{39}$$

g = any real number

n = any integer +, -, or 0

b_j = all real numbers (all but one can be zero).

This allows the input to include impulses, steps, ramps, sinewaves, decaying or increasing exponentials, $\frac{\sin at}{a}$, $\frac{\sinh at}{a}$, etc.

Considering the input current specified in equation (39), equation (28) becomes

$$\frac{\partial Z(s)}{\partial Z_b(s)} = \frac{D^2(s) I_b(s)}{N^2(s)} \tag{40}$$

$$= g^{-2} s^{-2n} \sum_{k=0}^{2m} b_k s^k I_b^2(s) \quad {}^2$$

$$= \sum_{k=0}^{2m} \left(b_k g^{-2} s^{k-2n}\right) I_b^2(s)$$

$$= \sum_{k=0}^{2m} \left(a_k s^{k-2n}\right) I_b^2(s) \quad . \tag{41}$$

The result is equation (35).

Consider the impedance of the elements. If the element is an inductor, then Z_b = sL and equation (28) may be written as

$$\frac{\partial Z(s)}{\partial (sL_b)} = \left(\frac{I_b(s)}{I(s)}\right)^2$$

$$\frac{\partial Z(s)}{\partial L_b} = s\left(\frac{I_b(s)}{I(s)}\right)^2 \tag{42}$$

If the element is a capacitor, then $Z_b = 1/sC_b$ and equation (28) becomes

$$\frac{\partial Z(s)}{\partial (1/C_b)} = \frac{1}{s}\left(\frac{I_b(s)}{I(s)}\right)^2 \quad . \tag{43}$$

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[2] The subscripts are changed because the b's are now the coefficients of the squared polynomial.

If the element is a resistor, equation (28) is

$$\frac{\partial Z(s)}{\partial R_b} = \left(\frac{I_b(s)}{I(s)}\right)^2 \quad . \tag{44}$$

Thus in general the sensitivity can be written as

$$\frac{\partial Z(s)}{\partial x_b} = s^j\left(\frac{I_b(s)}{I(s)}\right)^2 \quad . \tag{45}$$

where $j = +1$ if x_b is an inductor

$= 0$ if x_b is a resistor

$= -1$ if x_b is a compliance

Combining (45) and (41) yields the result

$$\frac{\partial Z(s)}{\partial x_b} = s^j \sum_{k=0}^{2m} \left(a_k s^{k-2n}\right) I_b^2(s)$$

$$= \sum_{k=0}^{2m} \left(a_k s^{k-2n+j}\right) I_b^2(s) \quad . \tag{46}$$

It can be shown that multiplication by s in the s - domain is equivalent to differentiation with respect to t in the time domain, that division by s in the s - domain is equivalent to integration in the time domain, and that multiplication of F(s) in the s - domain is equivalent to convolution in the time domain.[3] Thus, if the exponent of s in a term of equation (41) is greater than zero, the individual terms of (46) become

$$a_k \frac{\partial^{k-2n}}{\partial t^{k-2n}} \left[i_b(t) * i_b(t)\right] \quad .$$

If the exponent of s is negative, the term of (46) becomes

$$a_k \int \cdots \int \left[i_b(t) * i_b(t)\right] \quad .$$

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[3] See, for instance, Holbrook [Ref. 4] pp. 66, 68 and 108.

The number of integrations is equal to k-2n. The sensitivity, $\left(\frac{\partial Z}{\partial Z_b}\right)$, then becomes a sum of integrations and/or differentiations of the convolution of the current through the element of interest with itself, and equation (46) can be written as:

$$\frac{\partial z(t)}{\partial x_b} = \sum_{k=0}^{2m} a_k \frac{\partial^{k-2n+j}}{\partial t^{k-2n+j}} \left[i_b(t) * i_b(t) \right] .$$

This is equation (36).

To show the power relations of equation (37), Ohm's law is applied (see Figure 4). Again, unless otherwise stated, all computations are made in the s - domain.

$$\frac{\partial Z(s)}{\partial Z_b(s)} = \left[\frac{I_b(s)}{I_g(s)}\right]^2$$

$$E(s) = I_g(s)Z(s)$$

$$\partial E(s) = I_g \partial Z(s) + Z(s) \partial I_g(s)$$

$$\partial Z(s) = \frac{\partial E(s)}{I_g(s)}$$

Thus:

$$\frac{\partial E(s)/I_g(s)}{\partial Z_b(s)} = \left[\frac{I_b(s)}{I_g(s)}\right]^2$$

$$\frac{\partial E(s)}{\partial Z_b(s)} = \frac{I_b^2(s)}{I_g(s)}$$

$$\frac{\partial E(s)}{\partial Z_b(s)/Z_b(s)} = \frac{I_b(s)\ I_b(s)}{Z_b(s)\ I_g(s)} = \frac{E_b(s)I_b(s)}{I_g(s)} \tag{48}$$

If the input current is limited to an impulse in the time domain, $I_g(s)$ will be unity and equation (42) becomes

$$\frac{\partial E(s)}{\partial Z_b(s)/Z_b(s)} = E_b(s)I_b(s) \tag{49}$$

If a circuit is excited with an impulse current (see Figure 5), the ratio of the voltage across any element to the input current may be written as a transfer function such that:

$$\frac{E_b(s)}{I_g(s)} = T(s) \quad . \tag{50}$$

If the circuit is now excited by a current source equal to the current through the element of interest, the new element voltage, E(s), will be

$$E(s) = T(s)I_b(s) \quad . \tag{51}$$

This is true since the transfer function T(s) is a function of the network and not of the input current or voltage. The original input current was an impulse so that equation (50) can be written as $E_b(s) = T(s)$. Substituting this into (51) will yield

$$E(s) = E_b(s)I_b(s) \quad . \tag{52}$$

The input current can be written in the time domain as $i_b(t)$, and $E_b(s)I_b(s)$ can be written as $e_b(t) * i_b(t)$. Thus, (52) becomes in the time domain

$$e(t) = e_b(t) * i_b(t) \quad . \tag{53}$$

Equation (53) is the right hand side of equation (38). Thus, in order to measure the sensitivity of equation (38), the following procedure can be used. Excite the original circuit by an impulse and calculate $i_b(t)$. Then excite the circuit with $i_b(t)$. The voltage across the element is then equal to the sensitivity.

There are several interesting variations of equation (28). For instance: since $Y(s) = Z^{-1}(s)$

$$\frac{\partial Y(s)}{\partial Z_b(s)} = \frac{1}{Z^2(s)} \frac{\partial Z(s)}{\partial Z_b(s)} = -\frac{1}{Z^2(s)}\left[\frac{I_b(s)}{I(s)}\right]^2 = -\left[\frac{I_b(s)}{I(s)Z(s)}\right]^2$$

$$\frac{\partial Y(s)}{\partial Z_b(s)} = -\left[\frac{I_b(s)}{E(s)}\right]^2 \tag{55}$$

$$\frac{\partial Y(s)}{\partial Z_b(s)/Z_b(s)} = -Z_b(s)\left[\frac{I_b(s)}{E(s)}\right]^2 = \left[\frac{E_b(s)}{E(s)}\right]^2 \tag{56}$$

If I = EY is substituted into equation (28)

$$\frac{\partial Z(s)}{\partial Z_b(s)} = \left[\frac{I_b(s)}{I(s)}\right]^2 = \left[\frac{E_b(s)Y_b(s)}{E(s)\ Y(s)}\right]^2$$

$$\frac{Y^2(s)\partial Z(s)}{Y_b^2(s)\partial Z_b(s)} = \left[\frac{E_b(s)}{E(s)}\right]^2 .$$

Remembering that $Z = Y^{-1}$, the partial derivative yields $\partial Z = -\partial Y/Y^2$.

Putting this into (56) results in

$$\frac{\partial Y(s)}{\partial Y_b(s)} = \left[\frac{E_b(s)}{E(s)}\right]^2 \tag{57}$$

For another variation, $Z = Y^{-1}$ is used. Thus

$$\frac{\partial Z(s)}{\partial Y_b(s)} = -\frac{1}{Y^2(s)}\ \frac{\partial Y(s)}{\partial Y_b(s)} = -\frac{1}{Y^2(s)}\left[\frac{E_b(s)}{E(s)}\right]^2$$

$$\frac{\partial Z(s)}{\partial Y_b(s)} = \left[\frac{E_b(s)}{I(s)}\right]^2 \tag{58}$$

$$\frac{\partial Z(s)}{\partial Y_b(s)Y_b(s)} = -\frac{Y_b(s)E_b^2(s)}{I^2(s)} = -\frac{E_b(s)I_b(s)}{I^2(s)} \tag{59}$$

If e(t) is restricted to a delta function, then equation (55), (56), and (57) may be written as

$$\frac{\partial Y(s)}{\partial Z_b(s)} = -I_b^2(s) \tag{60}$$

$$\frac{\partial Y(s)}{\partial Z_b(s)/Z_b(s)} = -E_b(s)I_b(s) \tag{61}$$

$$\frac{\partial Y(s)}{\partial Y_b(s)} = E_b^2(s) \tag{62}$$

If i(t) is restricted to a delta, equation (58) and (59) may be written as

$$\frac{\partial Z(s)}{\partial Y_b(s)} = -E_b^2(s) \tag{63}$$

$$\frac{\partial Z(s)}{\partial Y_b(s)/Y_b(s)} = -I_b(s)E_b(s) \tag{64}$$

If the denominator of the right hand side of these equations are restricted to the forms of equation (39), time domain functions similar to that of equation (47) can be obtained. For any sensitivity calculation however, the impulse driving function gives the simplest and most direct result.

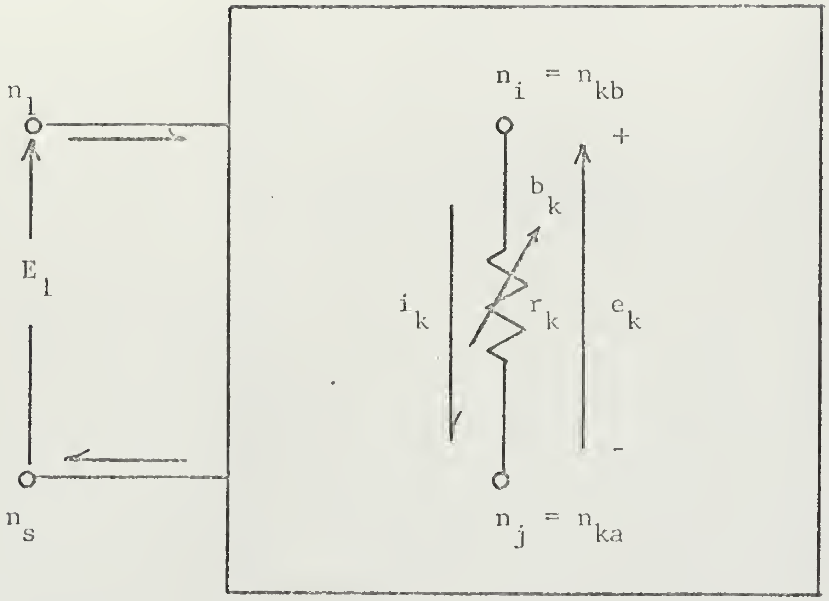


Figure 1

Two Terminal Network with Variable r_k
Used for Development of Cohn's Theorem

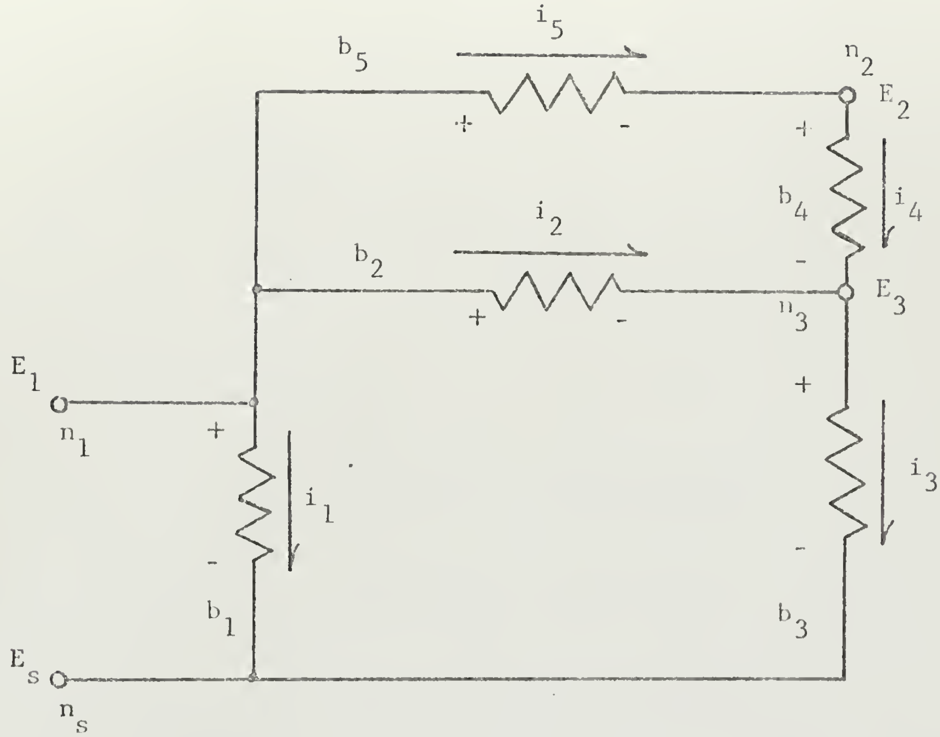


Figure 2

Five Branch Network Used to
Demonstrate Cohn's Theorem.

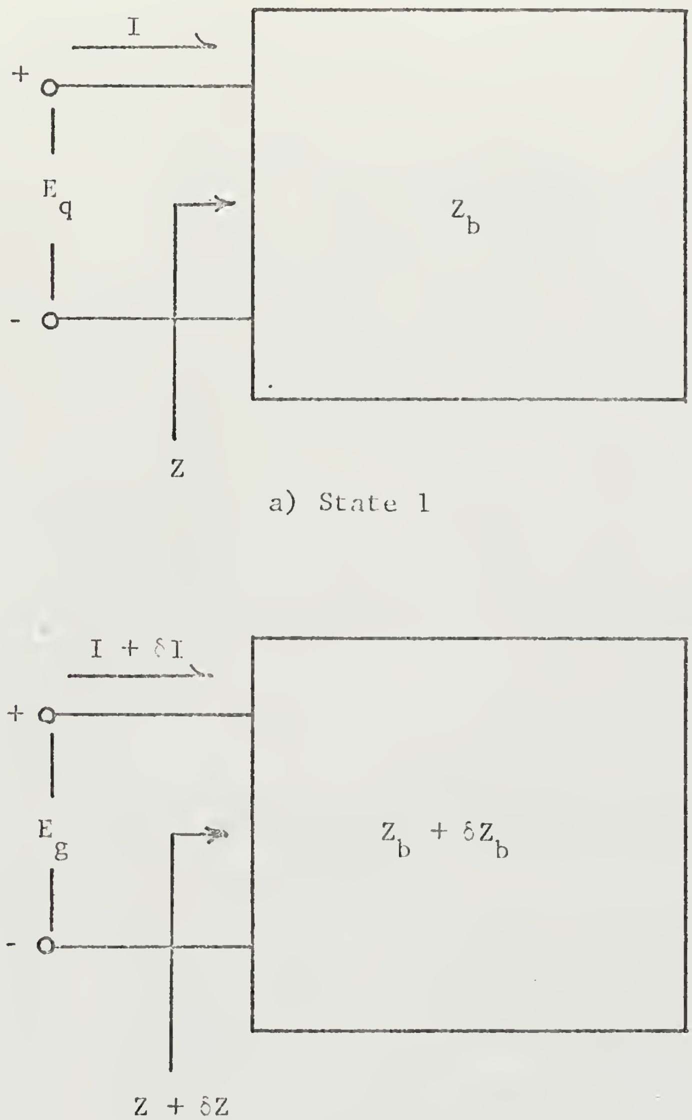


Figure 3

Two Circuits with the Same Topology
But with Different Internal Impedances.

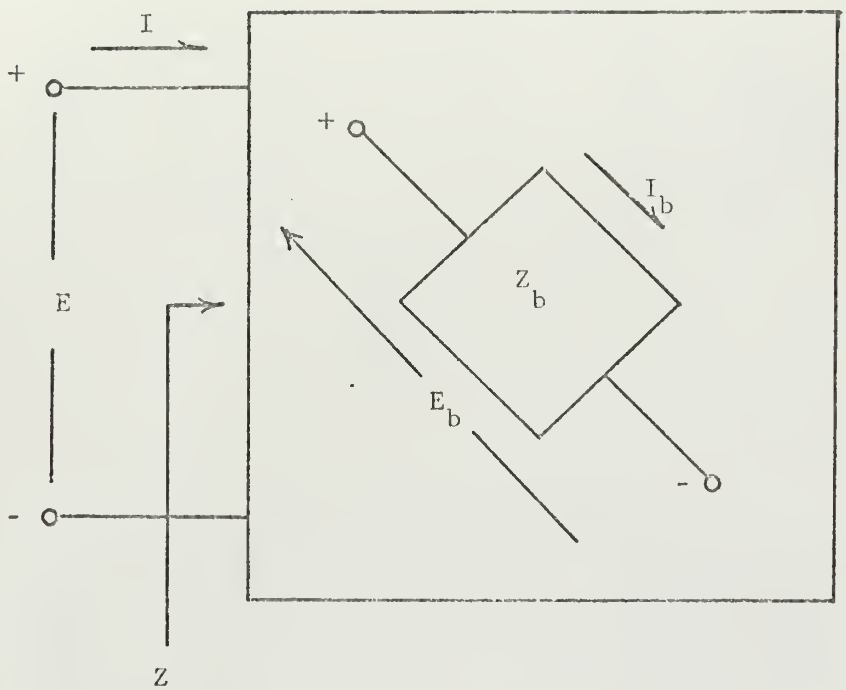
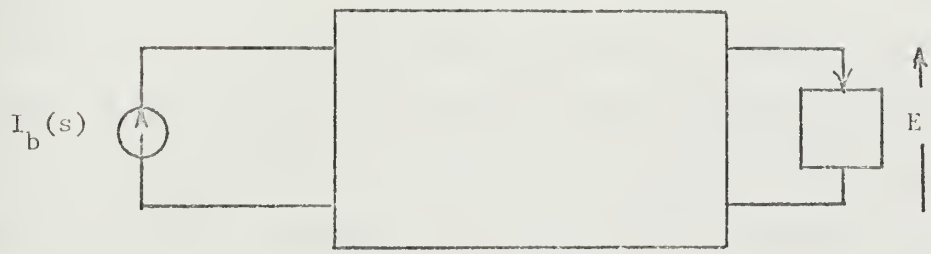


Figure 4

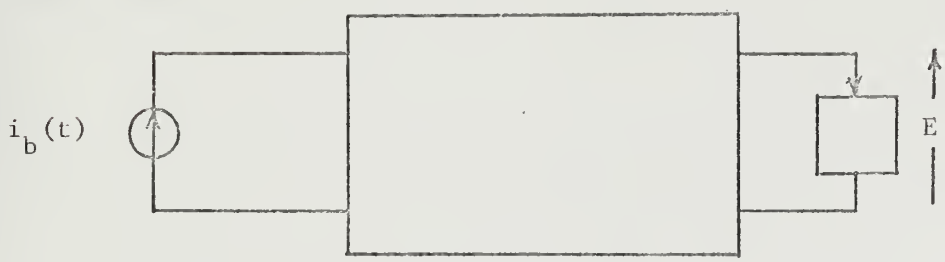
Two Terminal Networks Used for Developing the Power Relations.

If a constant input current is required, a current source is connected to the input terminals. If a constant voltage source is required, a voltage source is connected. In either case E = IZ.

$$\frac{E_b(s)}{I_g(s)} = T(s)$$



$$E = T(s)I_b(s)$$
$$= E_b(s)I_b(s)$$



$$E = i_b(t) * e_b(t)$$

Figure 5

Two Terminal Network Used for Developing the Time Domain Solution.

APPENDIX A

Example Using a Parallel RLC Circuit

For the circuit in Figure 6; let $R = 1.0$, $L = 10^{-4}$, $C = 10^{-7}$. Calculate the sensitivity of the input impedance with respect to each of the circuit components for a delta input current. Also, determine the sensitivity of the input voltage with respect to $\delta Z/Z$ for each component with the same delta input.

An impulse was approximated by a triangular pulse $8.0(10^{-7})$ seconds wide and $2.5(10^{6})$ amps at its center. This input current was used as data for an ECAP solution to arrive at the currents through and voltages across the components.

The data obtained from the ECAP solution was used as input data to Subroutine SEN which is in the Computer Program section of this paper. Since outputs were obtained from ECAP every 50 calculations, the time increment for SEN was $5(10^{-6})$. The final time was $5(10^{-4})$. The delta was chosen to give it unit area so COEF is 1.0. The Laplace transform of a delta is a constant. Thus, SN and IPR were set at zero. The IELEM and ITYPE values were determined by the type of element and type of sensitivity required.

The results are shown in Figures 7 through 12.

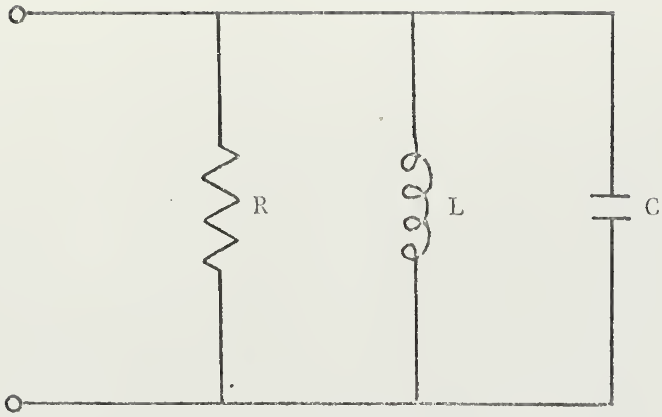


Figure 6

Parallel RLC Circuit Used for Sample Calculations.

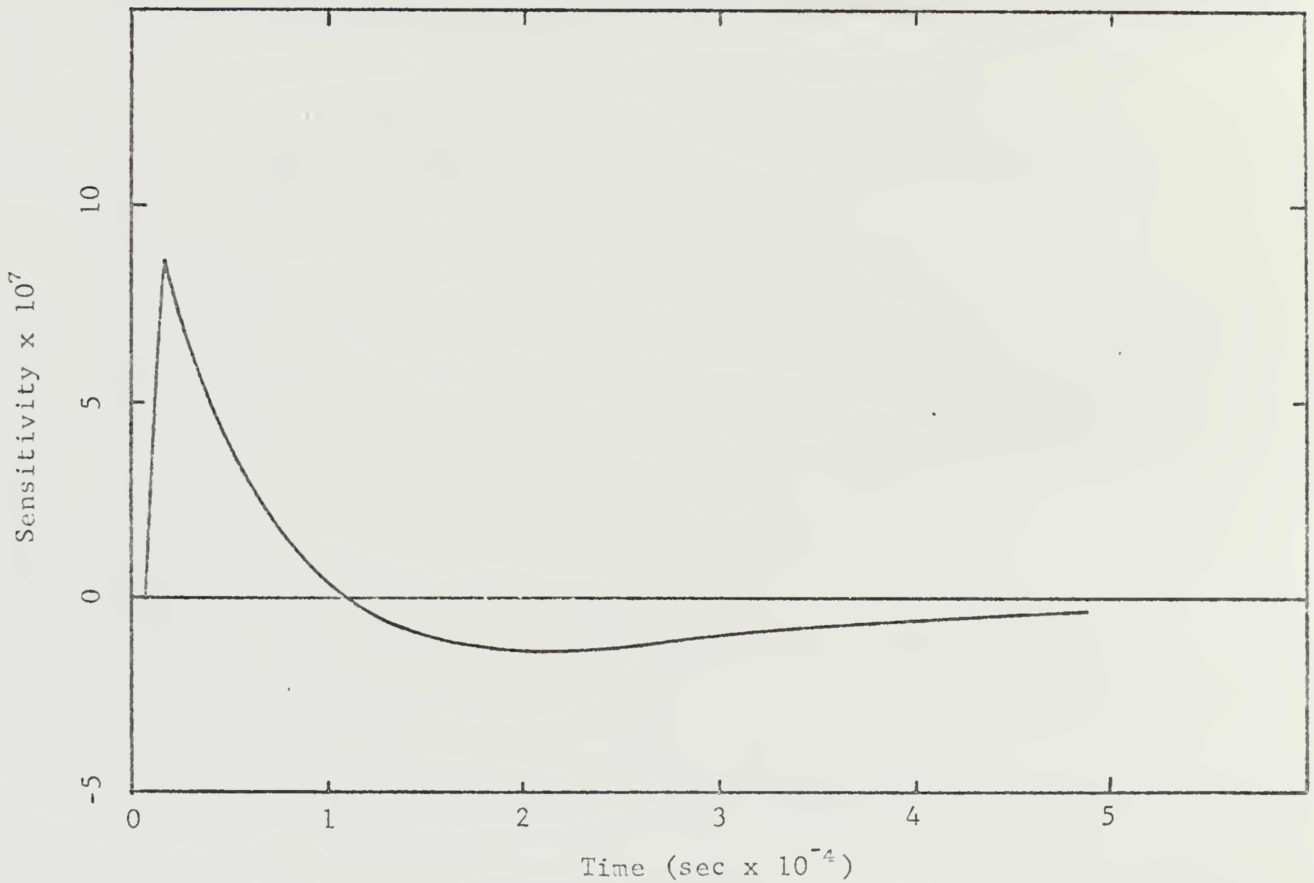


Figure 7 - Change in the input impedance with respect to a change in an internal inductance.

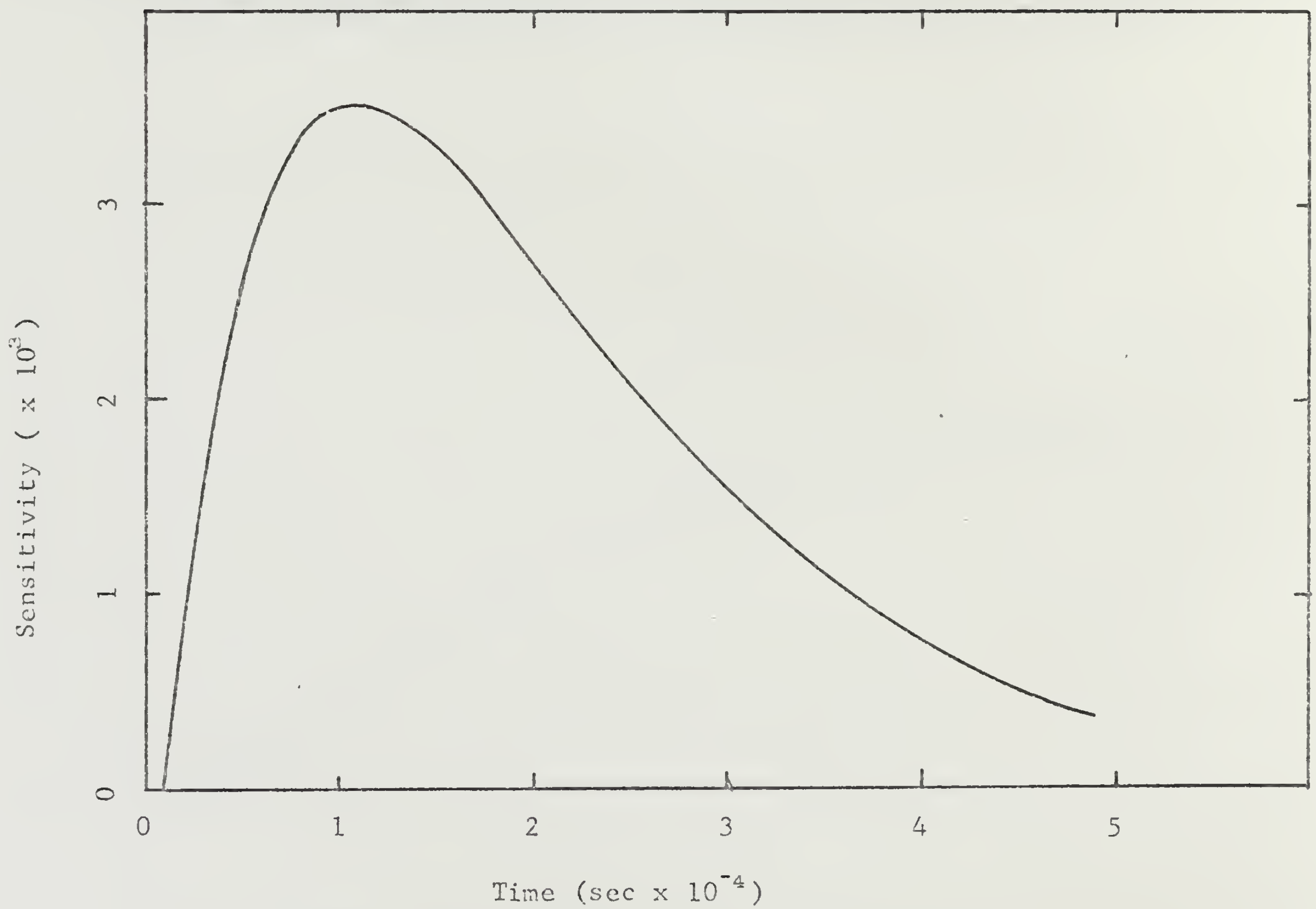


Figure 8 - Change in the input impedance with respect to a change in an internal resistance.

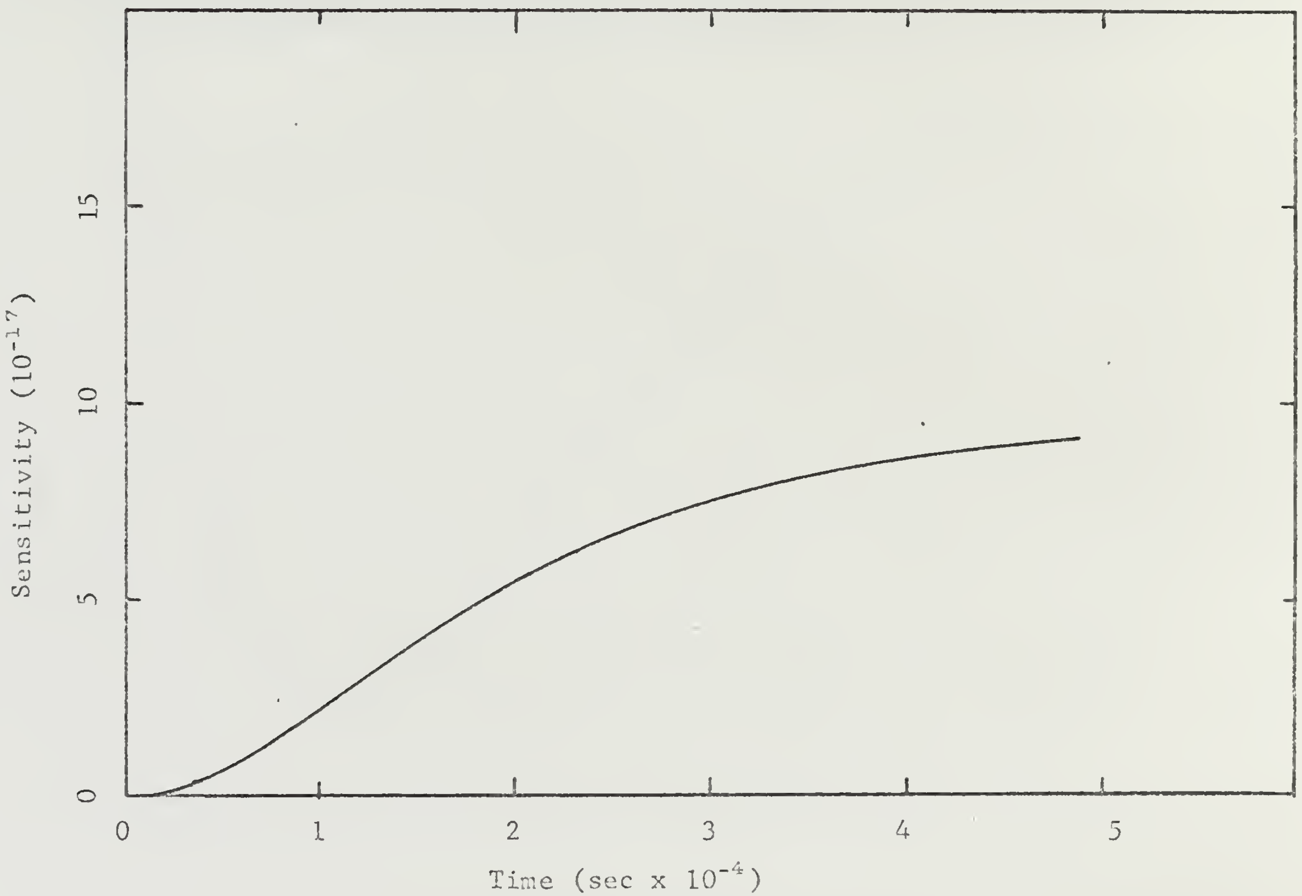


Figure 9 - Change in the input impedance with respect to a change in an internal reciprocal capacitance.

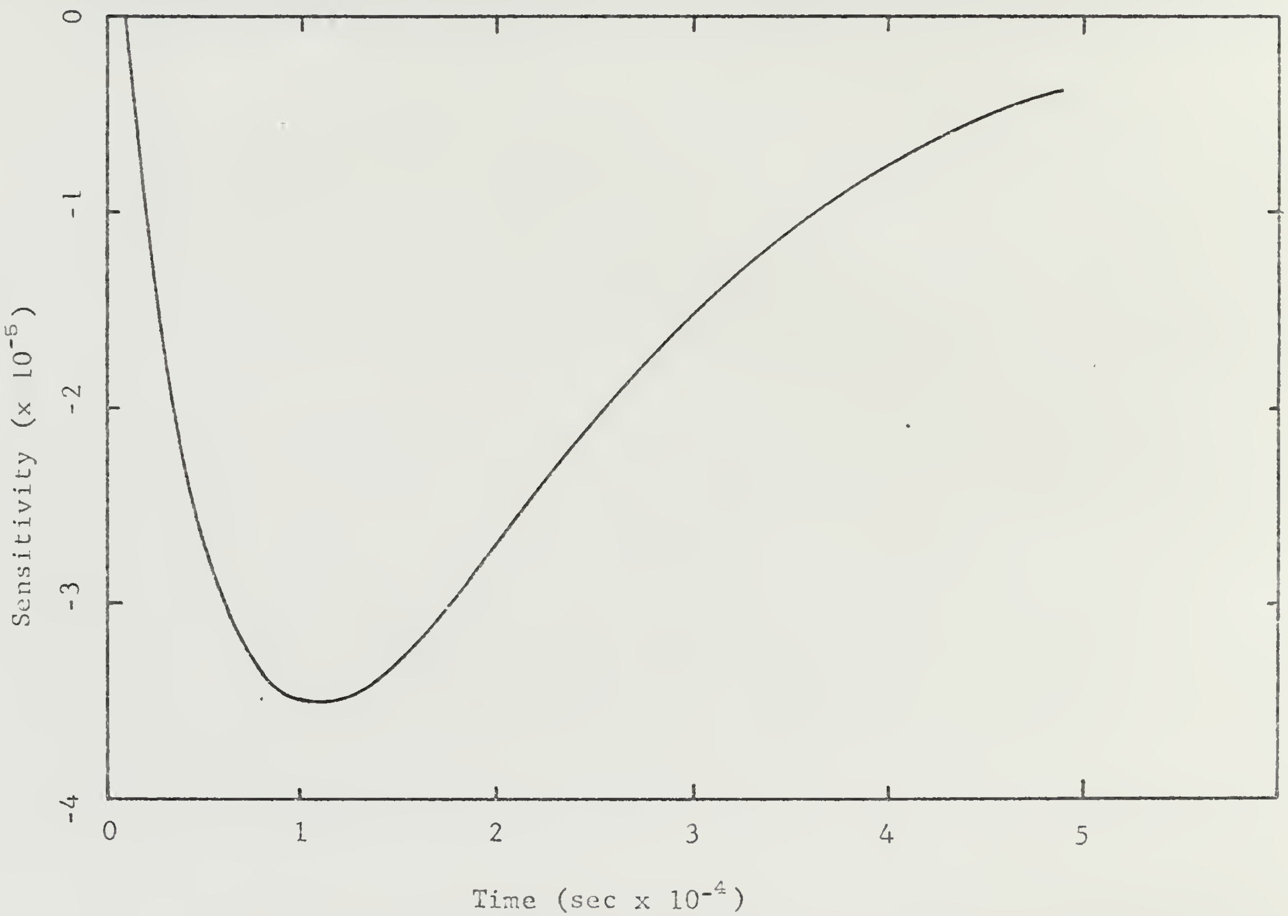


Figure 10 - Change in the input voltage with respect to a fractional change in reciprocal capacitance for an impulse input current.

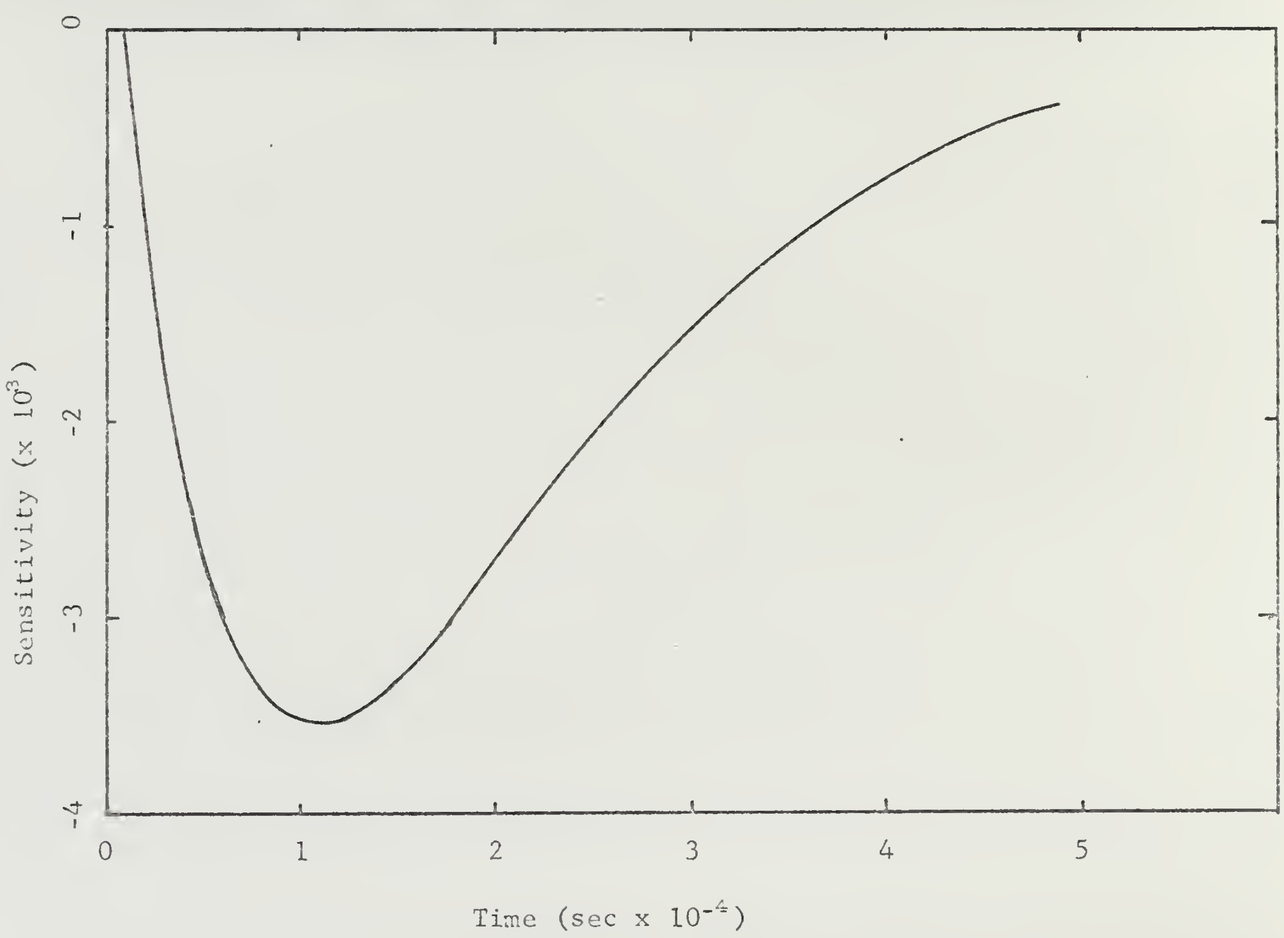


Figure 11 - Change in the input voltage with respect to a fractional change in resistance for an impulse input current.

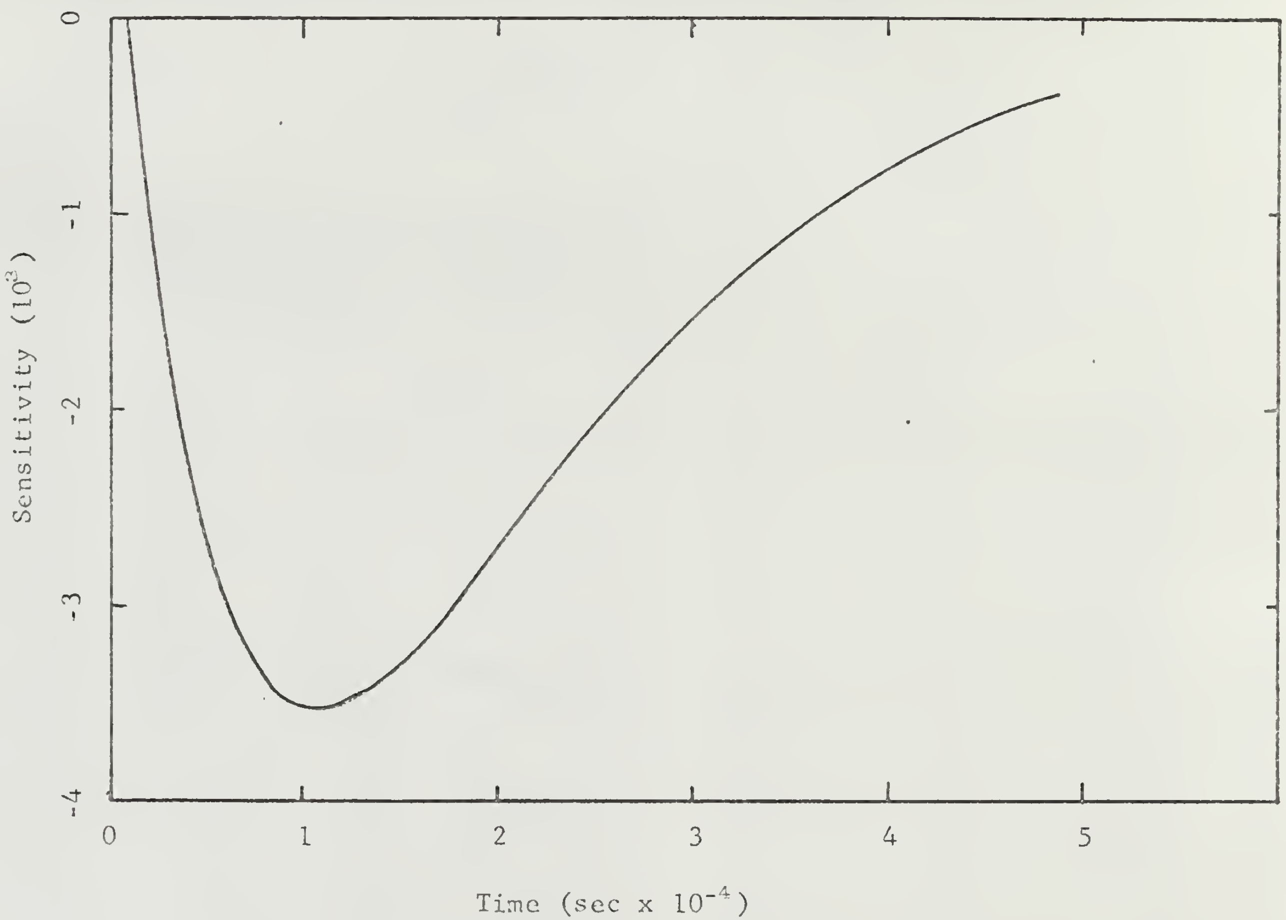


Figure 12 - Change in the input voltage with respect to a fractional change in inductance for an impulse input current.

COMPUTER PROGRAM

```
      SUBROUTINE SEN    (H,TF,COEF,SN,IPR,KEY1,IELEM,ITYPE,A,
     1B,E,T,SENS)
C
C   PURPOSE
C      TO COMPUTE THE SENSITIVITY OF THE INPUT IMPEDANCE
C      OR INPUT ADMITTANCE OF ANY RLC CIRCUIT WITH RESPECT TO
C         1. THE CHANGE IN ANY INTERNAL IMPEDANCE
C         2. THE CHANGE IN ANY INTERNAL ADMITTANCE
C         3. THE CHANGE IN ANY IMPEDANCE DIVIDED BY THAT
C            IMPEDANCE
C         4. THE CHANGE IN AN ADMITTANCE DIVIDED BY THAT
C            ADMITTANCE
C      THE SUBROUTINE WILL ALSO CALCULATE THE SENSITIVITY OF
C      THE INPUT VOLTAGE WRT DELTA Z OVER Z, WHERE Z IS ANY
C      INTERNAL IMPEDANCE. THE ELECTRICAL SOURCE MUST BE
C      EXPRESSED AS A LAPLACE TRANSFORM.
C
C   USAGE
C      CALL SEN(H,TF,COEF,SN,IPR,KEY1,IELEM,ITYPE,A,B,E,SENS)
C
C      TWO DATA CARDS ARE REQUIRED FOR THE GRAPH TITLE IF A
C      GRAPH OUTPUT IS REQUESTED. THE FIRST CARD MUST CONTAIN
C      THE FIRST 48 CHARACTERS OF THE TITLE AND THE SECOND
C      CARD THE SECOND 48.
C
C   DESCRIPTION OF PARAMETERS
C      H     - THE INCREMENT OF THE ARGUMENT VALUES
C      TF    - THE FINAL TIME OF THE INPUT VECTOR (A)
C      COEF  - VALUE OF THE SOURCE NUMERATOR COEFFICIENT
C      SN    - POWER OF THE S TERM OF THE SOURCE NUMERATOR
C      IPR   - HIGHEST POWER IN THE SOURCE NUMERATOR
C      KEY1  - CONTROLS GRAPH OUTPUT: 0 PRODUCES NO GRAPH
C      IELEM - IDENTIFIES THE ELEMENT OF INTEREST: ENTER A 1
C              FOR AN INDUCTOR, A 0 FOR A RESISTOR, OR A -1
C              FOR A CAPACITOR
C      ITYPE - TYPE OF SENSITIVITY REQUIRED.  ENTER A 0 IF
C              THE CONVOLUTION IS TO BE BETWEEN DIFFERENT
C              VECTORS.  ENTER A 1 IF THE CONVOLUTION IS TO B
C              BETWEEN IDENTICAL VECTORS.
C      A     - CURRENT OR VOLTAGE VECTOR FOR THE ELEMENT OF
C              INTEREST
C      B     - CURRENT OF VOLTAGE VECTOR FOR THE ELEMENT OF
C              INTEREST.
C      E     - COEFFICIENTS OF THE SOURCE DENOMINATOR IN
C              ORDER OF INCREASING POWER.
C      T     - OUTPUT VECTOR OF THE TIME
C      SENS  - OUTPUT VECTOR OF THE SENSITIVITY
C
C   REMARKS
C      1. THE EQUIDISTANT VECTOR OF THE ELEMENT CURRENT OR
C         VOLTAGE(A) MUST BE FURNISHED WHEN THE SENSITIVITY
C         SOLUTION REQUIRES THE SELF CONVOLUTION OF CURRENT
C         OR VOLTAGE. THE EQUIDISTANT VECTOR (B) MUST BE
C         FURNISHED ALSO BUT MAY BE ZERO UNLESS THE
C         SENSTTIVITY SOLUTION REQUIRES THE CONVOLUTION OF
C         DIFFERENT VECTORS. BOTH VECTORS MUST BE IN 5E15.7
C         FORMAT.
C      2. MAXIMUM LENGTH OF (A) AND (B) IS 100 POINTS.
C
C   SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C      DAHCO2 (H,TF1,TF2,A,B,D)
C      DET3 (H,Y,Z,NDIM,IER)
C       QSF (H,Y,Z,NDIM)
```

```
C
C     METHOD
C        SQUARES A POLYNOMIAL, EXTRACTS THE POWERS AND USES
C        THESE POWERS TO DETERMINE THE NUMBER OF DIFFERENTIA-
C        TIONS OR INTEGRATIONS TO BE PERFORMED.
C
C     REFERENCES
C        (1) JAMES G. HOLBROOK, LAPLACE TRANSFORMS FOR
C               ELECTRONIC ENGINEERS, PERGAMON PRESS, 1966.
C        (2) A.L. DAHLVIG,APPLICATIONS OF COHN'S SENSITIVITY
C               THEOREM TO TIME DOMAIN RESPONSES, MASTERS THESIS,
C               NAVAL POSTGRADUATE SCHOOL, 1970.
C
      DIMENSION A(100),B(100),C(100,100),D(201,2),Y(201),
     1Z(201),W(201),SENS(201),E(11),DEN(25),T(201)
      INTEGER  SD(25),SN
      REAL LAB1/4H    /
      REAL*8 ITITL(12)
C
      I2=TF/H
      I3=I2+I2+1
      SN=SN+SN
      IP1=IPR+1
      L=IPR+IPR+1
C
C        DETERMINE THE TYPE OF SENSITIVITY REQUIRED
C
      IF (ITYPE.EQ.0) GO TO 81
      DO 84 I=1,I2
      B(I)=A(I)
   84 CONTINUE
   81 CONTINUE
C
C        INITIALIZE DEN AND SD
C
      SD(1)=0
      DEN(1)=C.000
      DO 11 I=2,L
      DEN(I)=0.000
      SD(I)=I-1
   11 CONTINUE
C
C        SQUARE THE INPUT POLYNOMIAL
C
      DO 12 I=1,IP1
      DO 13 K=1,IP1
      J=I+K-1
      DD=E(I)*E(K)
      DEN(J)=DD+DEN(J)
   13 CONTINUE
   12 CONTINUE
      COEF=COEF*COEF
      DO 4 I=1,L
      IF(DEN(I).EQ.0.0000)GO TO 14
      GO TO 4
   14 SD(I)=0
    4 CONTINUE
      WRITE(6,906)
      WRITE(6,111) (I,DEN(I),SD(I),I=1,L)
C
C        PREPARE FOR CONVOLUTION
C
      TFZ=TF
C
C        CONVOLVE INPUT VECTORS; INITALIZE SENS
C
      CALL DAHCO2 (H,TF,TFZ,A,B,D)
      DO 16 I=1,I2
      T(I)=D(I,1)
      Y(I)=D(I,2)
      SENS(I)=0.00000
   16 CONTINUE
```

```
      DO 21 M=1,L
      J=SD(M)-SN
C
C        DETERMINE ELEMENT TYPE AND SET VALUE OF J
C
      IF (IELEM) 74,75,76
   74 J=J-1
      GO TO 75
   76 J=J+1
C
C        DETERMINE WHETHER INTEGRATION, DIFFERENTIATION OR
C        NO ACTION IS REQUIRED
C
   75 IF (J) 22,23,24
C
C        BEGIN REQUIRED INTEGRATIONS AND EVALUATE SENS
C
   22 J=-1*J
      CALL QSF(H,Y,W,I2)
      WRITE(6,907)
      IF (J.LT.2) GO TO 73
      DO 31 K=2,J
      CALL QSF(H,W,W,I2)
      WRITE(6,907)
   31 CONTINUE
      GO TO 73
   73 DO 41 I=1,I2
      SENS(I)=SENS(I)+(DEN(M)/COEF)*W(I)
   41 CONTINUE
      GO TO 21
C
C        NO ACTION IS REQUIRED, EVALUATE SENS
C
   23 DO 42 I=1,I3
      SENS(I)=SENS(I)+(DEN(M)/COEF)*Y(I)
   42 CONTINUE
      GO TO 21
C
C        BEGIN REQUIRED DIFFERENTIATIONS AND EVALUATE SENS
C
   24 CALL DET3(H,Y,W,I2,IER)
      WRITE (6,104) IER
      IF(J.LT.2) GO TO 99
      DO 32 K=2,J
      CALL DET3(H,W,W,I2,IER)
      WRITE (6,104) IER
   32 CONTINUE
   99 DO 43 I=1,I2
      SENS(I)=SENS(I)+(DEN(M)/COEF)*W(I)
   43 CONTINUE
   21 CONTINUE
C
C        DETERMINE ELEMENT TYPE FOR OUTPUT PURPOSES
C
      IF (IELEM)15,77,78
   15 WRITE(6,909)
      GO TO 64
   77 WRITE (6,910)
      GO TO 64
   78 WRITE (6,908)
   64 IF (KEY1.EQ.0) GO TO 79
      READ (5,911)(ITITL(I),I=1,12)
      CALL DRAW(I2,T,SENS,0,0,LAB1,ITITL,0,0,1,1,1,1,6,4,1,
     1LAST)
   79 CONTINUE
  101 FORMAT(5E15.7)
  104 FORMAT(5X,' IER = ',I5)
  111 FORMAT(3X,I3,3X,F15.7,6X,I3)
  906 FORMAT(3X,'TERM',5X,'COEFFICIENT',7X,'POWER')
  907 FORMAT(1H0,5X,'SPLINT')
  908 FORMAT(1H0,5X,'THE ELEMENT IS AN INDUCTOR')
  909 FORMAT(1H0,5X,'THE ELEMENT IS A CAPACITOR')
```

```
910 FORMAT(1H0,5X,'THE ELEMENT IS A RESISTOR')
911 FORMAT (6A8)
    RETURN
    END

    SUBROUTINE DAHC02 (H,TF1,TF2,A,B,D)
    DIMENSION A(100),B(100),C(100,100),D(201,2)
    N=TF1/H
    M=TF2/H
    NM=N+M+1
    DO 1 I=1,NM
  1 D(I,2)=0.
    DO 5 I=1,N
    DO 4 J=1,M
    C(I,J)=A(I)*B(J)
    K=I+J
  4 D(K,2)=D(K,2)+C(I,J)*H
  5 CONTINUE
    D(1,1)=0.
    DO 7 L=2,NM
    AA=L-1.
  7 D(L,1)=AA*H
    RETURN
    END

    SUBROUTINE DET3(H,Y,Z,NDIM,IER)
C
C
    DIMENSION Y(1),Z(1)
C
C       TEST OF DIMENSION
    IF(NDIM-3)4,1,1
C
C       TEST OF STEPSIZE
  1 IF(H)2,5,2
C
C       PREPARE DIFFERENTIATION LOOP
  2 HH=.5/H
    YY=Y(NDIM-2)
    B=Y(2)+Y(2)
    B=HH*(B+B-Y(3)-Y(1)-Y(1)-Y(1))
C
C       START DIFFERENTIATION LOOP
    DO 3 I=3,NDIM
    A=B
    B=HH*(Y(I)-Y(I-2))
  3 Z(I-2)=A
C       END OF DIFFERENTIATION LOOP
C
C       NORMAL EXIT
    IER=0
    A=Y(NDIM-1)+Y(NDIM-1)
    Z(NDIM)=HH*(Y(NDIM)+Y(NDIM)+Y(NDIM)-A-A+YY)
    Z(NDIM-1)=B
    RETURN
C
C       ERROR EXIT IN CASE NDIM IS LESS THAN 3
  4 IER=-1
    RETURN
C
C       ERROR EXIT IN CASE OF ZERO STEPSIZE
  5 IER=1
    RETURN
    END
```

```
      SUBROUTINE QSF(H,Y,Z,NDIM)
C
C
      DIMENSION Y(1),Z(1)
C
      HT=.3333333*H
      IF(NDIM-5)7,8,1
C
C     NDIM IS GREATER THAN 5. PREPARATIONS OF INTEGRATION
C     LOOP
    1 SUM1=Y(2)+Y(2)
      SUM1=SUM1+SUM1
      SUM1=HT*(Y(1)+SUM1+Y(3))
      AUX1=Y(4)+Y(4)
      AUX1=AUX1+AUX1
      AUX1=SUM1+HT*(Y(3)+AUX1+Y(5))
      AUX2=HT*(Y(1)+3.875*(Y(2)+Y(5))+2.625*(Y(3)+Y(4))+
     1Y(6))
      SUM2=Y(5)+Y(5)
      SUM2=SUM2+SUM2
      SUM2=AUX2-HT*(Y(4)+SUM2+Y(6))
      Z(1)=0.
      AUX=Y(3)+Y(3)
      AUX=AUX+AUX
      Z(2)=SUM2-HT*(Y(2)+AUX+Y(4))
      Z(3)=SUM1
      Z(4)=SUM2
      IF(NDIM-6)5,5,2
C
C     INTEGRATION LOOP
    2 DO 4 I=7,NDIM,2
      SUM1=AUX1
      SUM2=AUX2
      AUX1=Y(I-1)+Y(I-1)
      AUX1=AUX1+AUX1
      AUX1=SUM1+HT*(Y(I-2)+AUX1+Y(I))
      Z(I-2)=SUM1
      IF(I-NDIM)3,6,6
    3 AUX2=Y(I)+Y(I)
      AUX2=AUX2+AUX2
      AUX2=SUM2+HT*(Y(I-1)+AUX2+Y(I+1))
    4 Z(I-1)=SUM2
    5 Z(NDIM-1)=AUX1
      Z(NDIM)=AUX2
      RETURN
    6 Z(NDIM-1)=SUM2
      Z(NDIM)=AUX1
      RETURN
C     END OF INTEGRATION LOOP
C
    7 IF(NDIM-3)12,11,8
C
C     NDIM IS EQUAL TO 4 OR 5
    8 SUM2=1.125*HT*(Y(1)+Y(2)+Y(2)+Y(2)+Y(3)+Y(3)+Y(3)+
     1Y(4))
      SUM1=Y(2)+Y(2)
      SUM1=SUM1+SUM1
      SUM1=HT*(Y(1)+SUM1+Y(3))
      Z(1)=0.
      AUX1=Y(3)+Y(3)
      AUX1=AUX1+AUX1
      Z(2)=SUM2-HT*(Y(2)+AUX1+Y(4))
      IF(NDIM-5)10,9,9
    9 AUX1=Y(4)+Y(4)
      AUX1=AUX1+AUX1
      Z(5)=SUM1+HT*(Y(3)+AUX1+Y(5))
   10 Z(3)=SUM1
      Z(4)=SUM2
```

```
      RETURN
C
C     NDIM IS EQUAL TO 3
   11 SUM1=HT*(1.25*Y(1)+Y(2)+Y(2)-.25*Y(3))
      SUM2=Y(2)+Y(2)
      SUM2=SUM2+SUM2
      Z(3)=HT*(Y(1)+SUM2+Y(3))
      Z(1)=0.
      Z(2)=SUM1
   12 RETURN
      END
```

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13. ABSTRACT

A sensitivity theorem by R. M. Cohn states that for linear resistive circuits the ratio of a fractional change in the d.c. input resistance to a fractional change in an internal resistance is equal to the square of the ratio of the current through that internal resistance to the d.c. input current. The theorem is extended to show the sensitivity of input impedance to changes in internal impedances for an arbitrary network at all frequencies. Equations are developed which show the relation between sensitivities and instantaneous power in the frequency domain. An extension to the time domain makes digital computer solutions possible.

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c.1 Application of Cohn's
sensitivity theorem to
time domain responses.